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Fluid-Structure Interaction and Vortex Identification

J. Šístek[1,2], V. Kolář[3], F. Cirak[4] and P. Moses[2]

[1]Institute of Mathematics, Academy of Sciences of the Czech Republic
CZ-11567 Prague 1, Czech Republic

[2]Department of Mathematics, Faculty of Mechanical Engineering, Czech Technical University in Prague
CZ-12135 Prague 2, Czech Republic

[3]Institute of Hydrodynamics, Academy of Sciences of the Czech Republic
CZ-16612 Prague 6, Czech Republic

[4]Department of Engineering, University of Cambridge
Cambridge CB2 1PZ, United Kingdom

Abstract

Fluid-structure interaction is often associated with 3D large-scale vortical structures in the wake and a near-wake region which is strongly affected by the mean-flow shear. The present paper concentrates on the performance of vortex-identification schemes in the near-wake region of an inclined flat plate. Two widely used methods, the Q-criterion and λ_2-criterion, are compared with the triple-decomposition method (TDM) based on the extraction of a local shearing motion near a point. The following conclusion is drawn: to capture the vortical structures in the wake, both the Q-criterion and λ_2-criterion cannot—unlike TDM—avoid the bias in showing shearing zones, formed in the close proximity of the plate edges, as vortex regions.

Introduction

Many engineering applications represent internal or external fluid-structure interactions, including flows around bluff bodies. The resulting bluff-body wakes are usually characterized by 3D large-scale vortical structures in the near-wake region to be examined using different vortex-identification methods.

A direct comparison of two widely used vortex-identification methods, the Q-criterion [1] and λ_2-criterion [2], with the recently proposed triple-decomposition method (TDM) [3] is carried out for the vortex structure of a near-wake region of an inclined flat plate [4-8]. The latter method is based on the extraction of a local shearing motion.

The examined data sets describe the impulsively started incompressible flow around a flat plate (aspect ratio 2) at an angle of attack of 30°, Reynolds numbers Re=300 and 1200. Time instants with significant vortex structures suitable for this study were chosen from the unsteady simulation (after the plate travelled 8 and 14 chord lengths for Re=300 and 1200, respectively). This represents a model problem for fluid-structure interaction of an insect wing flapping at a high angle of attack, although the plate is considered rigid in the presented computation. Finite element method using unstructured mesh with 2.5M Taylor-Hood elements and 21M nodes was used to obtain the results in this paper.

There is a number of papers describing the flow around a flat plate at different Re and different incidence angles, e.g. [4-7] and a short review in [8]. Quite similar flow conditions as in the present paper (angle of attack 30°, Re=300, and aspect ratio of 2) have been assumed by Taira et al. [6, 7] showing a similar near-wake vortex structure as described here for the impulsively started flow around a flat plate. A common feature of the given flat-plate bluff-body wake close to the plate is (i) formation of separated (free) shear layers, namely leading-edge and trailing-edge "vortex sheets" which soon roll up to generate (ii) near-wake vortices. The pressure difference associated with lift generates around the plate tips of both sides (iii) characteristic wingtip vortices, that is long columnar vortices which are able to persist far downstream. These vortices may become particularly important in aerospace engineering studies dealing with different types of wingtips and wake vortex interactions.

The well-resolved separated thin shear layer originating from the leading edge of a plate, is sometimes labelled and interpreted as the so-called "vortex sheet". However, from the fluid-mechanical viewpoint, it is different quality than that of actual large-scale swirling motion of a typical tube-like vortex reached afterwards by rolling up of the shear layer due to Kelvin-Helmholtz instability. The response of two standard vortex-identification methods (Q, λ_2, see Appendix) towards the two different flow situations is to be examined and compared with the performance of the method TDM [3] (Appendix). The method aims at removing the biasing effect of a local shear, and this positive aspect of the TDM has been recently discussed in [9] on the background of other criteria.

Vortex Identification in the wake of an inclined flat plate

The vortex-identification results are summarized in Figures 1-4. Figures 1-2 deal with the flow at Re=300 and Figures 3-4 with the flow at Re=1200. The relative threshold levels (with respect to the maximum value within the examined domain) employed are very low, for the Q-criterion and λ_2-criterion are well below one percent and about three percent for the TDM residual vorticity. Apparently, the obtained vortex-identification results for the Q-criterion are quite close to those for the λ_2-method.

From Figures 1 and 3 it follows directly that an effort to capture the streamwise vortical structures in the wake downstream in a comparable manner with the residual vorticity of the TDM, both the Q-criterion and λ_2-criterion cannot—unlike TDM—avoid the bias in showing shearing zones, formed in the close proximity of the plate edges, as vortex regions. By taking a higher threshold value to diminish the shearing zones, the relevant streamwise vortices disappear completely as shown in Figures 2 and 4. This observation similarly holds for both Reynolds numbers under consideration.

The contour lines of vorticity component for the plate-symmetry plane, parallel with the coordinate plane x-y, are depicted including the identification results based on the TDM (colour) in Figure 5.

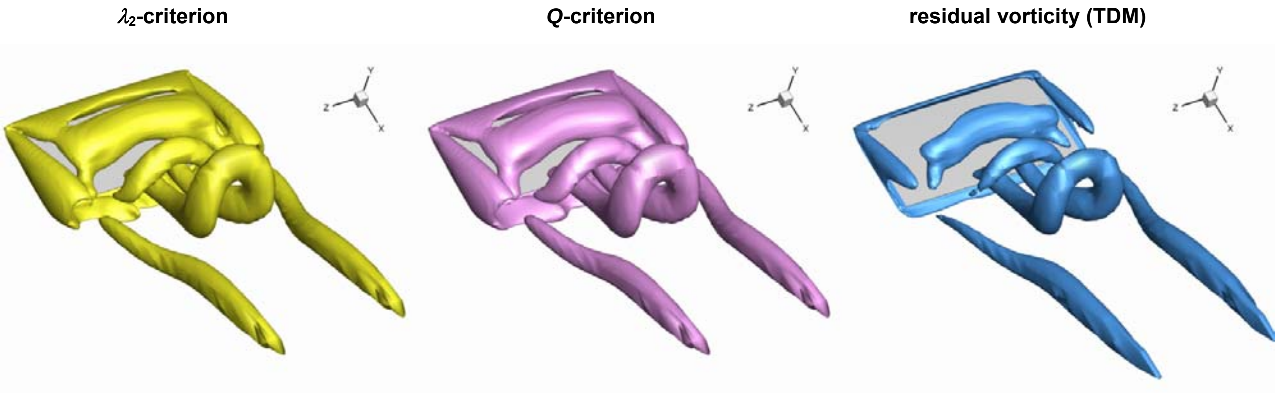

Figure 1. Results for the flow around a flat plate at Re=300 revealing a shearing bias of the λ_2-criterion and Q-criterion near the plate edges.

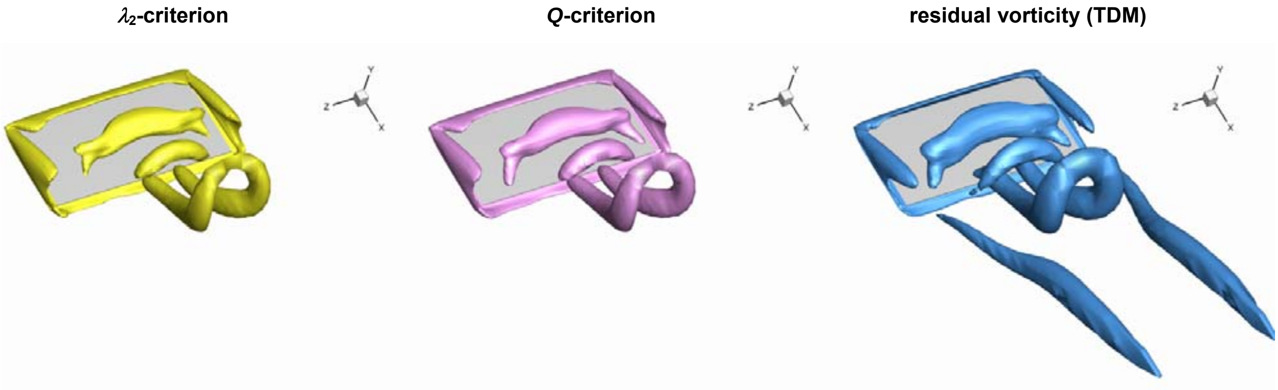

Figure 2. An attempt to diminish a shearing bias of the λ_2-criterion and Q-criterion near the plate edges by taking a higher threshold value (Re=300).

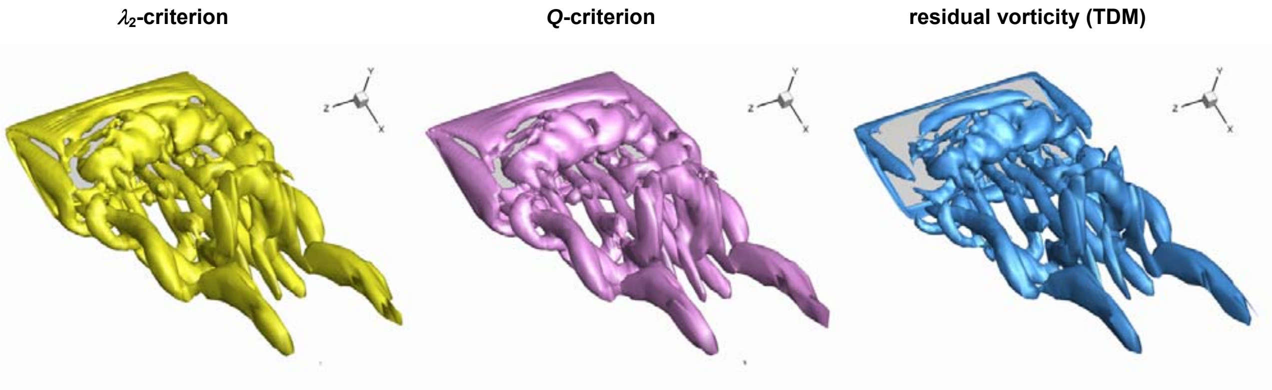

Figure 3. Results for the flow around a flat plate at Re=1200 revealing a shearing bias of the λ_2-criterion and Q-criterion near the plate edges.

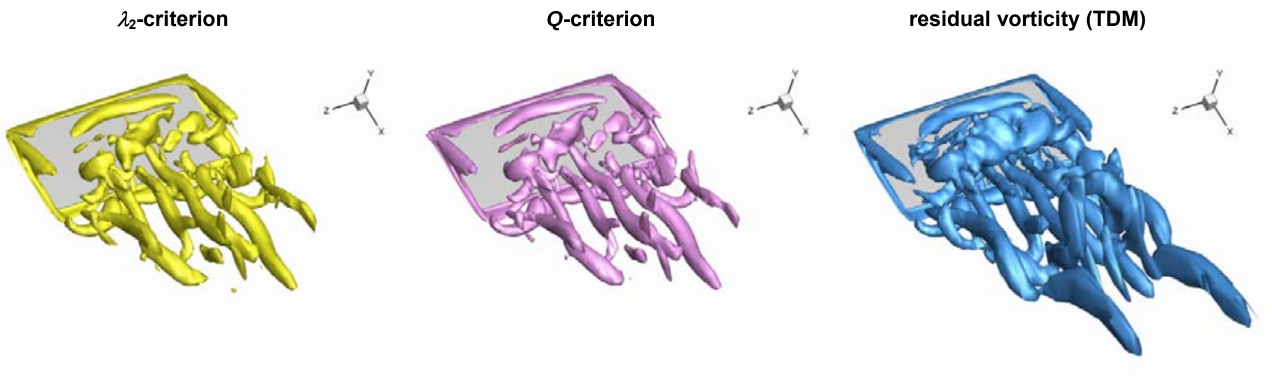

Figure 4. An attempt to diminish a shearing bias of the λ_2-criterion and Q-criterion near the plate edges by taking a higher threshold value (Re=1200).

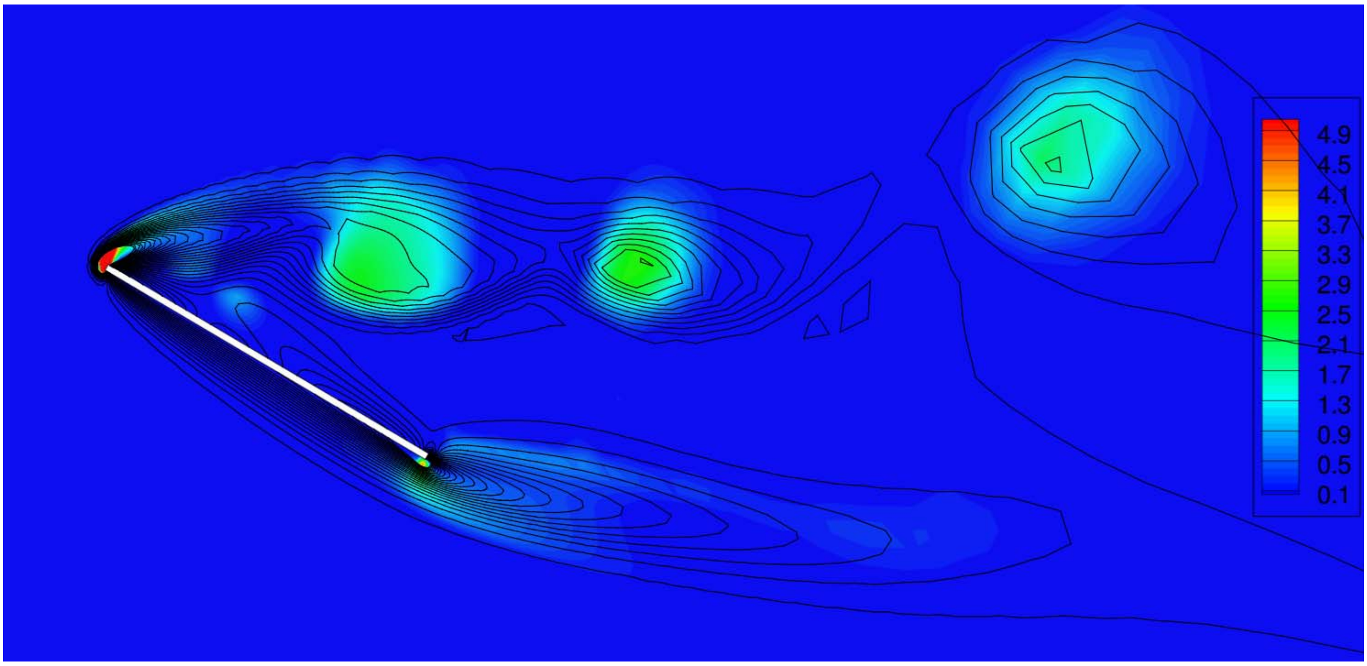

Figure 5. Vorticity (z-component) contour lines and vortex identification results (colour) using the magnitude of the TDM residual vorticity in the plane of a plate symmetry.

Discussion

Some previous applications of vortex-identification criteria to plate wake flows are as follows: the λ_{ci} (swirling-strength) criterion [10], based on the previous Δ-criterion [11], was employed in [4], and the Q-criterion was used in [6, 7]. It should be noted that the two criteria, Δ and λ_{ci}, are equivalent just (and only) for zero thresholds and that generally the Q-criterion is more restrictive than Δ-criterion. In the above mentioned applications [4, 6, 7], both criteria, λ_{ci} and Q, interpret free shear layers behind the plate edges as vortex regions.

The nature of separated (free) shear layers depends predominantly on the Reynolds number. At higher Reynolds numbers the so-called "shear-layer vortices", developing in the free shear layers located between the separation point and the first shed Strouhal vortex, may occur in the bluff-body near wake as discussed for a circular cylinder in [12]. These small-scale vortices develop due to the shear-layer Kelvin-Helmholtz instability and merge into the Strouhal vortices. The transitional region associated with the onset of shear-layer vortices was roughly estimated for a circular cylinder [12] in the Reynolds number range 1000-3000.

In the present case of an inclined flat-plate wake the Reynolds number was low and the developed shear layers were relatively short before rolling up into large-scale tube-like vortices (cf. Figures 1-5). Therefore, the interesting structural phenomenon of shear layer-vortices was not indicated and hence not taken into further considerations.

The observed bias of the two vortex-identification methods towards local shearing could be—very loosely said—, at least partially, attributed to a complex non-linear nature of the dependence of the vortex-identification criterial quantity on (total) vorticity which can absorb shearing effects "in a boundless manner". The latter fact represents an inherent property of vorticity. On the other hand, the TDM method somehow defines the local shear and directly removes it. However, all three methods remain local pointwise schemes unlike the examined non-local phenomenon of a vortex.

Conclusions

Three vortex-identification schemes (Q, λ_2, and TDM) have been applied to the near-wake region of an inclined flat plate. Their results have been compared by applying very low threshold values. The performance of two widely used methods, the Q-criterion and λ_2-criterion, indicates that these schemes are—in the close proximity of the plate edges—relatively shear-biased in comparison with the TDM which performs better in removing the biasing effect of a local shear.

Acknowledgments

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Appendix

Q-criterion [1]: Vortices of an incompressible flow are identified as connected fluid regions with a positive second invariant of the velocity-gradient tensor $\nabla \mathbf{u}$, $\nabla \mathbf{u} = \mathbf{S} + \mathbf{\Omega}$, $\mathbf{S}$ is the strain-rate tensor, $\mathbf{\Omega}$ is the vorticity tensor (in tensor notation below the subscript comma denotes differentiation),

$$Q \equiv \frac{1}{2}\left(u_{i,i}^2 - u_{i,j}u_{j,i}\right) = -\frac{1}{2}u_{i,j}u_{j,i} = \frac{1}{2}\left(\|\mathbf{\Omega}\|^2 - \|\mathbf{S}\|^2\right) > 0 \,. \tag{A.1}$$

This is fulfilled in the regions where the vorticity magnitude prevails over the strain-rate magnitude.

λ_2-criterion [2]: This criterion is formulated on dynamic considerations, namely on the search for a pressure minimum across the vortex. The quantity $\mathbf{S}^2 + \mathbf{\Omega}^2$ is employed as an approximation of the pressure Hessian after removing the unsteady irrotational straining and viscous effects from the strain-rate transport equation for incompressible fluids. A vortex region is defined as a connected fluid region with two negative eigenvalues of $\mathbf{S}^2 + \mathbf{\Omega}^2$, that is, if the eigenvalues are ordered, $\lambda_1 \geq \lambda_2 \geq \lambda_3$, by the condition $\lambda_2 < 0$.

Triple-decomposition method (TDM) [3]: The TDM is expressed through the corresponding triple decomposition of a local motion. As a result, $\nabla \mathbf{u}$ consists, unlike the double decomposition $\nabla \mathbf{u} = \mathbf{S} + \mathbf{\Omega}$, of three parts so that the strain-rate tensor $\mathbf{S}$ and vorticity tensor $\mathbf{\Omega}$ are cut down in magnitudes to "share" their portions through the third term $(\nabla \mathbf{u})_{\mathrm{SH}}$ associated with a local shearing motion. In terms of the *residual* parts of $\mathbf{S}$ and $\mathbf{\Omega}$ it reads

$$\nabla \mathbf{u} = \mathbf{S}_{\mathrm{RES}} + \mathbf{\Omega}_{\mathrm{RES}} + (\nabla \mathbf{u})_{\mathrm{SH}} . \qquad \text{(A.2)}$$

The first term on the RHS of (A.2) stands for an irrotational straining, the second one represents a rigid-body rotation. The third term of the triple decomposition denoted as $(\nabla \mathbf{u})_{\mathrm{SH}}$ and representing a shearing motion is described by a "purely asymmetric tensor" fulfilling in a suitable reference frame (the subscript comma denotes differentiation)

$$u_{i,j} = 0 \quad OR \quad u_{j,i} = 0 \quad (\text{for all } i, j). \qquad \text{(A.3)}$$

From the viewpoint of the double decomposition, $\nabla \mathbf{u} = \mathbf{S} + \mathbf{\Omega}$, the term $(\nabla \mathbf{u})_{\mathrm{SH}}$ itself is responsible for a specific portion of vorticity labelled "*shear* vorticity" and for a specific portion of strain rate labelled "*shear* strain rate" while the remaining portions of $\mathbf{S}$ and $\mathbf{\Omega}$ are labelled "*residual* strain rate" and "*residual* vorticity".

The TDM is closely associated with the so-called "basic reference frame" (BRF) where it is performed. The TDM results generated (i.e. separated) in the BRF are valid for all other frames rotated (not rotating) with respect to the BRF under an orthogonal transformation. In the BRF, **(i)** an effective shearing motion is shown "in a clearly visible manner" described by the tensor form (A.3) under the definition condition that **(ii)** the effect of extraction of a "*shear* tensor" is maximized within the following decomposition scheme applicable to an arbitrary reference frame

$$\nabla \mathbf{u} \equiv \begin{pmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{pmatrix} = \begin{pmatrix} residual \\ tensor \end{pmatrix} + \begin{pmatrix} shear \\ tensor \end{pmatrix} \qquad \text{(A.4)}$$

where the *residual* tensor is defined as

$$\begin{pmatrix} u_x & (\operatorname{sgn} u_y)\,\mathrm{MIN}(|u_y|,|v_x|) & \bullet \\ (\operatorname{sgn} v_x)\,\mathrm{MIN}(|u_y|,|v_x|) & v_y & \bullet \\ \bullet & \bullet & w_z \end{pmatrix} . \qquad \text{(A.5)}$$

The following notation is used in (A.4) and (A.5): u, v, w are velocity components, subscripts x, y, z stand for partial derivatives. The remaining non-specified pairs of off-diagonal elements of the *residual* tensor in (A.5) are constructed analogously as the specified one, each pair—if considered separately—being either symmetric or antisymmetric. The effect of extraction of the *shear* tensor is maximized by changing the reference frame under an orthogonal transformation so that the absolute tensor value of the *residual* tensor is minimized, or the closely related scalar quantity $|S_{12}\Omega_{12}| + |S_{23}\Omega_{23}| + |S_{31}\Omega_{31}|$ is maximized, as shown in [3]. This extremal condition guarantees that an effective shearing motion is recognized in the BRF as a third elementary part of the triple decomposition and can be extracted from $\nabla \mathbf{u}$ following (A.4) and (A.5). For details and the qualitative description of the flow kinematics near a point adopted in the TDM, see [3].

The *residual* vorticity tensor $\mathbf{\Omega}_{\mathrm{RES}}$ representing a rigid-body rotation is assumed to provide an "unbiased shear-free measure" of the actual swirling motion of a vortex. The magnitude of the residual vorticity is therefore employed in the TDM-based vortex identification as shown in the present contribution and in [13].

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